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IN JUNIOR AND SENIOR HIGH SCHOOLS

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THE MATHEMATICS TEACHER

VOLUME XIX OCTOBER, 1926 NUMBER 6

REVISION OF COLLEGE PREPARATORY MATHEMATICS

By MARIE GUGLE
Assistant Superintendent of Schools, Columbus, Ohio
and President of the National Council of Teachers of Mathematics

College entrance mathematics is a variable quantity; until recently each college set its own entrance requirements. As President Butler said, they "were going their several ways with sublime unconcern for the policies of other colleges, for the needs of secondary schools, or for the general public interests. . . . No secondary school could adjust its work and its program to their requirements."

The last twenty-five years have seen several attempts, not only toward a more accurate definition of college entrance requirements, but also toward a reorganization of the teaching of mathematics according to the pupils' requirements. These attempts are most interestingly presented in the First Yearbook of the National Council of Teachers of Mathematics, just off the press.

Today, more than ever before, junior and senior high schools are offering opportunities for their pupils to get an excellent preparation in this subject. For decades the traditional course was a year and a half of algebra, with many impractical topics, and a year and a half of plane and solid geometry. The attempted revisions of this traditional course included the elimination of obsolete topics, the reduction in the number of theorems and the elimination of incommensurables. Still the results were unsatisfactory.

The development of the junior high school gave an opportunity for teachers to work out something different, a complete elementary cycle, including arithmetic, intuitive geometry, and algebra, woven together into a unified whole that gives interest and meaning to the work.

The National Committee on Mathematical Requirements gave its sanction to this type of mathematics in its Report of 1920,

Read before the Educational Conference of Ohio State University, April 9, 1926.

and laid down the thesis that the type of mathematics that is best suited to the needs and capacities of boys and girls of junior high school age is the best possible college preparatory mathematics. Unfortunately this thesis has not been fully accepted by some senior high school teachers and by more college instructors.

Most of you are familiar with the eliminations and substitutions recommended by the Committee. These have been worked into well organized courses in a number of series of junior high school texts. Incidentally, the methods of teaching this reorganized mathematics have been greatly improved. The value of learning arithmetic, algebra, and geometry in their relations to one another cannot be too highly stressed.

Sheets have been distributed outlining the geometric facts and relationships which junior high school pupils have learned intuitively or experimentally that make such a solid background for demonstrative geometry in the tenth grade that the senior high schools should capitalize it. (See appendix.)

If plane and solid geometry could be taught in a year and a half to pupils who began it in the tenth grade wholly ignorant of all this intuitive geometry, surely to those who already have this knowledge demonstrative geometry should be taught in one year only.

Senior high schools do not dare attempt this revision as long as colleges insist for college entrance requirements on one year of plane and a half-year of solid geometry.

This suggestion is the one of those made by the National Committee that can be most readily followed. Fortunately the requirements of the College Entrance Examination Board, as revised in 1922-23, have followed the recommendations of the National Committee, not only as to the elimination of topics in algebra and the substitution of more worthwhile numerical trigonometry, but also as to the acceptance of one unit of mathematics covering both plane and solid geometry.

The Entrance Board makes the following recommendations:

1. That the number of required theorems be reduced. Only eighty-nine are included in plane geometry and of these only about a third are required for examination purposes.
2. That a year's course in plane and solid geometry replace the required unit of plane geometry.

What are the advantages of such a course? The usual complaint from college professors that pupils are not prepared comes from their lack of familiarity with algebra, whose last acquaintance for many was three long years before. Is it not more or less unreasonable to expect the knowledge to carry over such a period? How many of you could read a page of Greek or Latin three or four years after your graduation if you had not looked at it in the meantime? Not many. Besides you had the advantage of added maturity.

If the majority of pupils elect only two or even three units of preparatory mathematics, one and a half units for geometry leaves too small a part for algebra, which is of more use in later mathematics. The part of geometry most used has been learned intuitively in the junior high and the knowledge is therefore more permanent.

If the time in the senior high school spent on demonstrative geometry is reduced, more time is available for algebra and for algebra applied in trigonometry.

The unified or general mathematics, as reorganized for junior high schools, is far more satisfactory and effective than the old plan of arithmetic in grades seven and eight and algebra in the ninth. Some attempts have been made to work out a general course for the senior high schools, but without success when demonstrative geometry was included. Intuitive geometry for the junior high school fuses beautifully with arithmetic and algebra; but demonstrative geometry is logic and will not fuse. If, however, this subject is offered in the tenth grade as a unit course covering both plane and solid, there is left a wonderful opportunity in the eleventh and twelfth grades for working out a unified course, including algebra, trigonometry, and some analytics and calculus, any part or all of which would more closely articulate with the beginning college mathematics than the present course possibly can, that is, the course in which solid geometry is the last phase of high school mathematics.

However, not all the reorganization needs to be done in the secondary schools. We are willing and anxious to do our share. But we expect the college instructors to meet us half way. Too many of the latter are trained in mathematics but are not trained to teach. They even ridicule the idea that they need to know

how to teach. While I believe in thorough training in subject matter, I think it is possible for a college professor to know so much mathematics that he loses all appreciation of the difficulties of beginning trigonometry for a freshman who has been unfortunate enough not to start the subject in high school. Too many professors go on the "sink or swim" plan. I maintain that the beginning of any subject, whether it is started in the elementary, junior or senior high school, or in college, must be started in a simple meaningful way. Trigonometry and logarithms can be presented to ninth grade pupils in such a way that they will appreciate and enjoy them. Yet they are a pons assinorum to many a freshman in college.

Some teachers in senior high schools imitate college professors in wanting to make pupils feel that their subject is "hard." Little do they realize the effect of emotion or attitude on learning. Many a mother has said to a child, "Well, I never could learn arithmetic, and I don't suppose you can." That child never does.

When I taught geometry in high school, I deliberately planned during the first few weeks to break down the prejudices that pupils brought with them. After a few weeks, I have had more than one pupil say; "Oh! I didn't know geometry was like this. I was told it was *hard*." No pupil with a mind set against a subject will ever learn it.

At the recent N. E. A. meeting in Washington, Dr. Briggs of Columbia gave one of the most significant discussions of the convention, which I was unfortunate enough to miss hearing, but about which I heard much favorable comment. His topic was "A Neglected Phase of Education" or "Emotionalized Attitudes." If the address is available in printed form, it would be worth our careful study.

If a pupil is to learn anything quickly and well, he must have the right attitude or mind-set towards his teacher, his school, his subject, his fellows, and his own ability. College professors cannot afford to ignore these conditions for learning any more than secondary teachers.

Freshman college students have another serious difficulty, that of adjusting themselves to a different type of text book. Writers of texts in the elementary and secondary fields have given more

consideration to learning processes and have written their books for the pupils. College texts are mainly encyclopedias of subject matter. The authors of some seem to take special delight in making the subject far more abstruse than necessary. Added to that is the attitude of many college professors that it is beneath their dignity to teach freshmen. So what ought one to expect of them? I pity the college freshman.

In summary, what can the secondary schools do?

1. Teach reorganized mathematics, including intuitive geometry in all seventh, eighth, and ninth grades, whether organized as junior high schools or not.
2. Teach plane and solid demonstrative geometry as a year's course.
3. Give in the eleventh grade a combined or general course based on trigonometry and algebra, but using geometric relations.
4. For pupils who elect mathematics in the twelfth grade, offer a second unit of this general course, including more of analytics and the calculus.

What can the colleges do?

1. Since many entrants come with only two units of mathematics taken two years before graduations, the colleges should offer freshman courses similar to the eleventh and twelfth year work in the high schools.
2. For certain colleges, as the engineering, where more training is desirable, a definite number of required units for these colleges should be fixed and their freshman work organized accordingly.
3. They should encourage entrants to offer one unit of junior high school mathematics, one unit of plane and solid geometry, and at least one unit of algebra and trigonometry, as a combined course.
4. They should be very familiar with the degree of difficulty of topics in algebra as recommended for high schools by the National Comimttee on Mathematical Requirements and not ridicule and discourage pupils who cannot do the more complicated problems that are now eliminated from modern texts.

5. They should rewrite their college texts so that their subjects are presented in a psychological and pedagogical manner and articulate with the best modern high school texts.
6. The persons chosen to teach the freshman should be real teachers, artist teachers, not mere mathematicians. These teachers should be able to transmit to their students a real appreciation and enthusiasm and a genuine love for the subject, which something bigger and greater than mere manipulation. Mathematics is the foundation of the earth and the heavens and without it was not anything made that was made. It is the law by which God wrought and by which man achieves in his steady march toward greater civilization.

Appendix

Geometric Forms, Facts, and Symbols Taught In Junior High Schools Usable Foundation for Demonstrative Geometry In Tenth Grade

I *Construction and interpretation* of line, bar, and circular graphs.

II *Concepts of*: edge, straight edge, parallel, English measuring units, common metric units, length, width, surface, area, squared paper, bisection of lines, angles and classification as to sizes, angle measures, reading angles, a protractor and its use, perpendicular lines, horizontal lines, vertical lines, oblique lines, square, rectangle, perimeter, triangle, hexagon, cube, volume, rectangular prism, dimensions, lateral faces, substituting numerical values, parallelogram, quadrilateral, rhombus, reading geometric figures, complementary angles, exterior angles of a triangle, classification of triangles as to size and form, hypotenuse, circle, diameter, radius, circumference, intersection, base angle, vertex angle, arc, transversal, corresponding angles, vertical angles, alternate interior and exterior angles, et al, supplementary angles, median, ratio, proportion, quadrant, sextant, similar figures, transit, polygon, trapezoid, base, altitude, regular polygon, pentagon, octagon, cylinder, cylindrical surface, pi, pyramid, cone, sphere.

III Symbols for:

Parallel, square, rectangle, triangle, angle, perpendicular, parallelogram, arc, circle, perimeter, area, volume, sides of right triangle, etc.

IV Theorems, Postulates, Assumptions, and Constructions.

1. Usual postulates and assumptions.
2. Sum of angles of a triangle.
3. Exeterior angle of triangle equals sue of interior angles at the other vertices.
4. Formulas for areas of square, rectangle, triangle, parallelogram.
5. Constructions:
   - Various shaped triangles
   - Perpendicular lines
   - Bisection of lines and angles
   - Geometric designs
   - Parallel lines
   - To divide a line into any number of equal parts
   - Various quadrilaterals
   - Drawing to scale
   - Graphs of ratios
   - Regular polygons
6. Base angles of isosceles triangle are equal.
7. Facts about bisector of vertex angle of isosceles triangle, perpendicular from vertex to the base, and perpendicular bisector of the base.
8. Pythagorean theorem and its use in calculations.
9. Geometric representatives of the square roots of 2, 3 and 5.
10. Statements of angle relations in figure of two parallels cut by transversal.
11. If two parallels cut off equal parts on one transversal, they cut off equal parts on every transversal.
12. Relations and effects of diagonals in various quadrilaterals.
13. Ratios and their equality.
14. Geometric representations of algebraic expressions.
15. Formula for diagonal of square in terms of its side.

16. Similar figures and their proportions applied to problems.
17. Area of trapezoid.
18. Sum of interior angles of common regular polygons.
19. Formulas for circumference and area of circle and their use in calculations.
20. Formula for lateral surface and volume of a cylinder.
21. Location of points by co-ordinates and graph of linear equations.
22. Graph of quadratics.
23. Graph of sine, cosine, and tangent angles from 0° to 90°.
24. Congruence of triangles.
    (a) two sides and included angle respectively equal
    (b) two angles and included side respectively equal
    (c) three sides respectively equal.
25. In an isosceles triangle, angles opposite equal sides are equal.
26. Congruence of right triangles.
27. If two parallel lines are cut by a transversal, the alternate interior angles are equal.
28. Some idea as to what geometry is about.

THE TEACHING OF SECONDARY MATHEMATICS IN HOLLAND

D. J. E. SCHREK
Utrecht (Holland)

I. *Introduction.* When some months ago the Editor-in-Chief of this magazine asked me to write an article on the teaching of secondary mathematics in the Netherlands, I have with pleasure accepted his proposal. Indeed I think it to be very difficult for a foreigner to become acquainted with our system of secondary education in general and with the teaching of mathematics in particular.

On the occasion of the World's Fair at Brussels in 1910 the Dutch Government issued a book[1] on education in Holland. It was undoubtedly valuable, but of course it is now quite out of date. I am informed by the Dutch Department of Education that no other work of this kind has been published since that time. As to secondary mathematics we have the report[2] of the International Commission on the Teaching of Mathematics, issued by the Dutch sub-committee. This booklet, too, is out of date and has not been followed by another of the same kind. As the different American reports[3], which treat of Holland, have been based on the work of the International Commission abovementioned, the same is true for these. Since about 1920 many changes have taken place.

Dr. W. Lietzmann, the well-known German pedagogue and collaborator of the late Prof. Felix Klein, visited Holland in March, 1925, and gave a short but clear account of his journey in the Deutsches Philologen-Blatt[4]. The same author wrote in his magazine an article on some recent Dutch mathematical textbooks.[1]

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[1] Het onderwijs in Nederland. Groningen. Wolters. 1910. (411 pp.). It was also available in a French translation.

[2] Rapport sur l' Enseignement mathématique dans les Pays-Bas, publié par la Sous-Commission nationale de la Commission Internationale de l' Enseignement Mathématique. Delft. Waltman. 1911. (151 pp.)

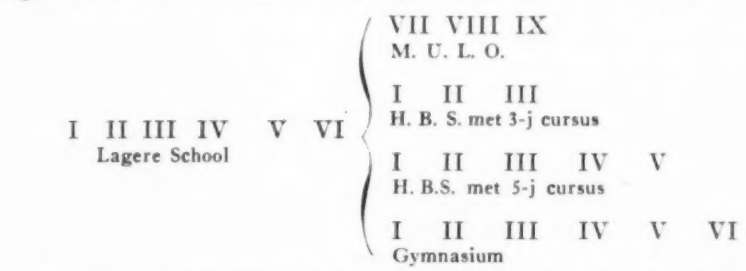
[3] e.g.: J. C. Brown. Curricula in Mathematics (U. S. Bureau of Education, Bulletin 1914, No. 45) and R. C. Archibald: The Training of Teachers of Mathematics (ibid., Bulletin 1917, No. 27).

[4] W. Lietzmann, Das holländische Schulwesen, insbesondere seine Reformanstalten. Deutsches Philologen-Blatt. 33 Jahrgang 1925. pp. 260-261.

[5] W. Lietzmann, Einige neuere mathematische Schulbücher aus Holland. Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht. 56 Jahrgang, 1925. pp. 47-49.

A more detailed list of Dutch textbooks composed by the author of this paper will be found in the second edition of the first part of Dr. Lietzmann's work on the teaching of mathematics.[1]

II. *Types of Schools.* Before discussing the teaching of mathematics in our country I wish first to indicate the different types of schools. After leaving the Elementary School ("Lagere School," 6 years) the pupil may follow a three-years course of a more advanced instruction ("Meer uitgebreid Lager Onderwijs," M. U. L. O.), which includes mathematics and foreign languages, but is still considered to be elementary. Secondary schools, though different in many respects, are however of two chief types, the Higher Burgher School ("Hoogere Burgerschool," usually denoted as H. B. S.) and the Gymnasium. The former non-classical schools are divided into two kinds: those with a three-year course and those with a five-year course. The latter is always a *humanist* gymnasium with Latin and Greek, for a school like the German Realgymnasium with Latin but without Greek, though often planned, does not exist as yet in Holland. We thus have the following general scheme:


Age: 6-7 7-8 8-9 9-10 10-11 11-12 12-13 13-14 14-15 15-16 16-17 17-18
I II III IV V VI
Lagere School
VII VIII IX
M. U. L. O.
I II III
H. B. S. met 3-j cursus
I II III IV V
H. B.S. met 5-j cursus
I II III IV V VI
Gymnasium

There are at present only 21 H. B. S. with a three-year course in Holland[2], many of them having been transformed into schools with a five-year course, while several of the still existing serve as a base for commercial schools. They have a limited curriculum

[1] W. Lietzmann, Methodik des mathematischen Unterrichts. Erster Teil. 2 Aufl. Leipzig. Quelle und Meyer, 1926. pp. 334-339.

[2] Many of the data and numbers mentioned in this article have been taken from the Verslag van den Staat van het onderwijs in het Koninkrijk der Nederlanden. (Report on the state of education in the Kingdom of the Netherlands), an official report annually offered to the States General by the Minister of Education. The last volume that has appeared is that of 1922-1923.

in mathematics and will not be further considered here; nor will be mentioned the numerous vocational (agricultural, commercial, technical) schools.

As to the founding of secondary schools we have to distinguish *public, private* (secular) and *denominational* schools. The public schools are established by the state or by the municipality, the private and denominational (both Protestant and Roman Catholic) by some society. In nearly all schools there is co-education, i.e. every school for boys is also open to girls, but there are separate schools for girls only. An entrance-examination is not required if the pupil has obtained from the head-master of the elementary school a certificate of sufficient ability.

1. *The H. B. S. with a five-year course.* Until some years ago there was only one kind of H. B. S. with a five-year course; in their curriculum mathematics and science were strongly emphasized. Though intended by their founder—Minister Thorbecke in 1863—as a school conferring an all-round education, they have proved to give an excellent preparation for higher technical and medical studies. At present the diploma of this H. B. S.—now called H. B. S. B.—entitles the pupil to go up for the examinations and graduation at the Technical Academy at Delft, the Agricultural Academy at Wageningen, the Commercial Academy at Rotterdam and the scientific and medical departments (including the veterinary at Utrecht) of the University.

In the last years, however, a second kind of H. B. S., the H. B. S. A., or literary-economic school, has been established. In these schools the languages (French, German and English, as in *every* secondary school in Holland), economy and bookkeeping are stressed. Those who have successfully passed the final examination may enter the Commercial Academy at Rotterdam and the commercial department of the University at Amsterdam, further rights not yet having been granted. In the fourth and fifth classes of these schools no, or very little, mathematics is taught.

On Jan. 1, 1923, there were 113 H. B. S., viz. 49 state-schools, 30 municipal and 34 private and denominational schools. To many of them an A-section has lately been connected, while several schools, formerly with a three-year course, have been transformed into independent H. B. S. A. with a five-year course.

2. *Secondary schools for girls only.* In the special H. B. S. with a five-year course for girls (secondary schools for girls) much emphasis is laid on a thorough study of modern languages, needlework, history of art and hygiene also being taught. The curriculum and final examination however are not regulated by law, so that no rights are granted to those, who have successfully passed the final examination of these in many respects excellent schools. This disadvantage has induced the authorities of several larger towns to connect these schools with a separate section of *four classes,* following on the second class, according to the scheme:

$$\text{I} \quad \text{II} \begin{cases} \text{III} & \text{IV} & \text{V} \\ \text{III} & \text{IV} & \text{V} & \text{VI} \end{cases}$$

In this second section the pupils are prepared for the final examination of the H. B. S. B.; owing to the fact that in this manner girls study *six* years on the same subjects that boys learn in *five,* the former may be instructed in a more quiet manner and in a slower pace, which is important from a hygienic point of view. There are as yet 10 of such H. B. S., attended by those girls, who wish to prepare themselves for higher studies.

3. *Gymnasia.* The gymnasia have a six-year course; in the first four classes the instruction is the same for all pupils, the fifth and sixth being divided into a section A and a section B. In these classes too, many lessons are given still in common, but while in A the classical studies are predominant the pupils in B apply themselves especially to mathematics and science. Both the A- and B-diploma entitle the pupils to study law and to attend the Agricultural Academy at Wageningen and the Commercial Academy at Rotterdam. Besides, the A-diploma gives admission to the study of theology and philology, while the B-diploma gives claim to that of medical and veterinary sciences, mathematical and physical sciences (including pharmacy) and engineering. On Jan. 1, 1923, there were in our country 34 public and 16 denominational gymnasia.

3. *Lycea.* The term "lyceum" has not been mentioned till now. What is a lyceum in Holland? Let us first say that it is not, as in Germany, a secondary school for girls and that the

term does not at all occur in the education-act. According to the law there exist *only* H. B. S. and gymnasia. But many have felt the disadvantage of a boy of 12 years old being obliged to make his choice as to his future education; indeed, because of the classical languages in the gymnasium a transition from the H. B. S. to that school in after years is difficult and the contrary too has many objections. In order to supply this much needed want the lyceum has been founded, a school in which the first two years are in common for all pupils. From the third class there are two sections, the gymnasium-section and the H. B. S.-section, each preparing for the usual final examination of these schools. As the law allows a change in the distribution of the lessons over the different classes this arrangement is possible; it is clear, however, that the accumulation of Latin and Greek in the middle classes (the classical studies beginning only in the third class) is a serious drawback for the gymnasium. After the founding of the first school of this type in 1910 (the "Nederlandsch Lyceum" in the Hague) many of them have been established. On Jan. 1, 1923, there were 23 lycea, two municipal, four private (secular) and 17 denominational. Three lycea were intended for girls only. It will be necessary to say that in the numbers mentioned previously for H. B. S. and gymnasium these lycea were *not* included.

The head of a H. B. S. is called director, that of a gymnasium rector. For the rector of a gymnasium the degree of doctor of philology is required; in case of absence his place is taken by a con-rector, a doctor of philology or of science. A lyceum is usually administered by a rector, a director (for the H. B. S.) and a con-rector (for the gymnasium).

The Dutch secondary schools are by far less great than those in the United States. On Jan. 1, 1923, the greatest gymnasium (the first municipal in the Hague) had only 422, the greatest H. B. S. (the second of these schools in Rotterdam) only 416 pupils.

After this rather long explanation we have still to mention a separate examination, the so-called "Staatsexamen," intended to those who, without having attended a gymnasium wish to obtain a diploma equivalent to that of these schools. It is held yearly in July and August in Utrecht.

III. *Subject matters of mathematical teaching according to the official curricula.* Having exposed in outline the Dutch system of secondary education let us now consider the mathematical teaching. It follows from what has been said already that we are chiefly concerned in the H. B. S. B. with a five-year course and the gymnasium, the lyceum being a combination of both.

As to the H. B. S. B. with a five-year course the Royal Decree of 16 June, 1920, is as yet operative. According to this Decree the curriculum of mathematics consists of Arithmetic, Algebra and Geometry. In Arithmetic (in the first two classes only) are taught, besides a recapitulation of the elementary-school subjects: proportions, calculation of square roots and operations with approximate numbers. In Algebra: positive and negative numbers, operations with monomials and polynomials, factoring, equations of the first degree with one or more unknowns, fractions, reduction of surds, fractional and negative exponents, logarithms, arithmetic and geometric series, compound interest, quadratic equations and higher equations, which can be solved with quadratics, graphs, logarithmic and exponential equations, recapitulation. Geometry includes plane and solid geometry, plane trigonometry and descriptive geometry (as far as the sphere).

The list of the numbers of weekly lessons (each of 50 minutes) is the following:

Classes	I	II	III	IV	V	total
Number of lessons	6	6	6	4	4	26

As has already been said the distribution of the lessons over the classes may be changed by permission of the Minister of Education, in the gymnasium as well as in the H. B. S.

For the H. B. S. A. with a five-year course the Royal Decree of 13 November, 1923, requires a total of 15 lessons without any further indication. In most cases mathematics is not taught in the highest class, nor is the pupil examined in this branch in his final examination.

The curriculum of the gymnasium is indicated by the Royal Decree of 7 June, 1919, which has caused many changes; the table of lessons is lastly modified by the Decree of 26 August, 1921. According to these regulations the mathematics in the

gymnasia consists of Arithmetic, Algebra and Geometry, as in the H. B. S. In Arithmetic and Algebra are taught in the first four classes: the operations with whole and fractional numbers and algebraical expressions, divisibility of numbers, proportions, equations of the first degree with one or more unknowns, surds, fractional and negative exponents, solving of simple quadratics, computations with logarithms, graphs; in the fifth and sixth classes a more detailed treatment of quadratic equations and a recapitulation of algebra. In Geometry: in the first four classes plane geometry and elementary trigonometry of the right-angled triangle; in the fifth and sixth classes solid geometry and recapitulation of plane geometry. These subjects are taught to *all* pupils. Moreover those of the B-section have to learn arithmetic and geometric series, logarithms, plane trigonometry, analytic geometry (including conic sections), the elements of the calculus, recapitulation and applications.

The numbers of weekly lessons are:

Classes	I	II	III	IV	V	VI	Total
Numbers of lessons	5	4	2	2	2(A) 5(B)	2(A) 5(B)	17(A) 23(B)

In the lycea these numbers are of course so modified that the first two classes are identical. The denominational lyceum at Zeist for instance, consisting of a H. B. S.-section A and B, and a gymnasium A and B, has the scheme:

Classes	I	II	III	IV	V	VI	Total	
Numbers of lessons	6	5	6*	—	—		17	H.B.S.A.
			6	4	5		26	H.B.S.B.
			2	2	2	2	19	Gymn. A
					5	4	24	Gymn. B

*In the literary-economic Section A two of these 6 lessons are devoted to commercial arithmetic and bookkeeping.

From the foregoing we may conclude that both in H. B. S. and gymnasium algebra, plane and solid geometry, and plane trigonometry are taught. Descriptive geometry (a term, which always means orthogonal projection on two planes and never central projection) is found solely in the H. B. S., while analytic geometry and the calculus are taught in the gymnasium only. Spherical Trigonometry is not decreed for either school at present, nor are a synthetic treatment of conic sections and an introduction into recent geometry.

IV. *The practice of mathematical teaching in Holland. Its development and improvement. Influence of modern tendencies.* Nobody, and certainly not the author of this paper, will be of opinion that with mentioning the official curricula all is said. As important as these is the manner in which they are interpreted by the teachers of a country.

It will surely be known to all American teachers how about 1900 the so-called Reform-movement has set in, both in France and in Germany. In the former the new Plan d' Etudes of Minister Leygues (1902) introduced many improvements in mathematical teaching, in which the function concept was strongly emphasized. Scholars like Emile Borel and Jules Tannery wrote textbooks, which nowadays are still worth reading. In Germany, it was above all Felix Klein, who devoted himself to the improvement of the teaching of mathematics. To this great mathematician of Göttingen, who died some months ago, we certainly are greatly indebted for the development of modern tendencies in Germany and abroad. Repeatedly he pointed out how the function concept should be the kernel of our mathematical teaching, how it should "imbue it like a leaven," and famous are his lectures on elementary mathematics from a higher point of view.[1] Famous too are the Meran Curricula of 1905, improved and re-edited in 1922.[2] Not less important was the work of the International Commission on the Teaching of Mathematics, founded at Rome in 1908 at the suggestion of Professor David Eugene Smith, the well-known historian and pedagogue of Teachers College, Columbia University, in New York[3]).

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[1] Felix Klein, Elementarmathematik vom höheren Standpunkte aus. 3 Aufl., besorgt von F. Seyfarth. Erster Band: Arithmetik, Algebra, Analysis (1924); Zweiter Band: Geometrie (1925). Leipzig. Teubner.

[2] Neue Lehrpläne für den mathematischen und naturwissenschaftlichen Unterricht an den höheren Lehranstalten, nach den Meraner Lehrplänen von 1905 neubearbeitet. Leipzig. Teubner. 1922.

[3] The history of the German Reform-movement has been written by Rud. Schimmack (Die Entwicklung der mathematischen Unterrichts-reform in Deutschland. Leipzig. Teubner. 1911); after Schimmack's premature death in 1912 his work has been continued by H. Weinreich (Die Fortschritte der mathematischen Unterrichts-reform in Deutschland seit 1910. Leipzig. Teubner. 1915). In the last edition of the first part of Klein's Elementarmathematik, just mentioned, Dr. Seyfarth has completed this history up to our days.

When we ask which subjects have been emphasized by the Reform-movement we may say that it is in the first place the function concept, this concept not being treated in an appendix, but penetrating the whole mathematical teaching (not only the algebraic). This was already demanded in the Meran Curricula and now again this claim is heard in "The Reorganization of Mathematics in Secondary Education," the important report[1] that has been issued by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America. The treatment of graphs and the function concept should have its natural conclusion in the elements of the calculus. Demonstrative geometry should be preceded by a short course of intuitive geometry (or propaedeutic course, as it is called in Germany). The logical training should not be neglected, but stress ought to be laid on intuition. Measuring and estimating not only in the class-room, but also in the courtyard of the school and in the field with primitive instruments and the theodolite were recommended; attention was asked for geometrical drawing and the use of the slide rule.[2] Artificial and unnatural applications should be avoided, but from physics, technics and everyday life examples should be borrowed. Required was too an incidental treating of the historical development of mathematics.

What of all this has been realized in secondary education in Holland? It is, in the first place, remarkable that very few teachers in the Netherlands are well-acquainted with foreign tendencies and claims so that the development of our teaching has taken place almost independently of other countries. Of course there were and there are prominent teachers who introduced into their lessons the same subjects as the Reform-movement demanded. Besides, some teachers were very well acquainted with the tendencies abroad. About 1905 two of them, F. J. Vaes and C. A. Cikot, demanded insistently introduction of the calculus into the secondary schools of Holland, but in vain.

[1] See pp. 64-73 of this report. The proposals may also be found in the Summary of the Report (pp. 53-60), issued by the U. S. Bureau of Education (Bulletin 1921, No. 32).

[2] A report on this question and a survey of what had been done in this respect in different countries was presented by Prof. D. E. Smith to the Congress of Cambridge (1912) of the International Committee on the Teaching of Mathematics (Publications du Comité Central, 2^{me} série, fasc. 1, p. 67).

When in 1921 in the medico-scientific congress at Utrecht the question was once more treated only a few seemed to know what had happened in the meantime in other countries[1] and the opposition was still great. The function concept has at present been introduced into the teaching both in H. B. S. and gymnasium, yet in the latter more than in the former. In the H. B. S. the functions $y = ax + b$ and $y = ax^2 + bx + c$, as well as their graphic representation seem to be universally introduced; graphs of simultaneous linear equations are drawn everywhere and extreme values of the quadratic function are studied. Less general seem to be $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, $y = x^3$, $y = \frac{ax + b}{cx + d}$, the exponential and logarithmic functions. The author of this paper is accustomed to explain the notion of inverse functions; I think this usually to be done in the gymnasia, perhaps less in the H. B. S. Only in the gymnasia seem to be studied such fractional functions as $y = \frac{ax^2 + bx + c}{dx^2 + ex + f}$ and their graphs of so many different forms. But evidently it is difficult to assert such things in general as they depend wholly upon the personal ideas of the teacher. One thing however is certain: all these subjects have occurred in the examination papers of the gymnasium and the "Staatsexamen" for several years, while those of the H. B. S. are still old-fashioned. The want of improvement in the H. B. S. teaching is evidently felt, as appears from a semi-official committee lately being constituted (chairman Dr. H. J. E. Beth at Deventer) for the purpose of making proposals in this respect.

The calculus is only taught in the gymnasium, but in the H. B. S. it seems to be used in a greater or less extent in mechanics in exposing the notions of rate and acceleration. Generally only the elements are taught: differentiation of simple functions (except the exponential and logarithmic usually), applications on maxima and minima, second differential coefficient, inflexion-points, easy integrations, applications in computing areas and volumes. Some teachers however go farther and treat

[1] In the Congress of Paris (1914) of the International Committee on the Teaching of Mathematics Prof. E. Beke of Budapest gave an extensive account of this question (Publication du Comité Central, 2me série, fasc. III, p. 59).

infinite series and Taylor's and Maclaurin's theorems. In analytic geometry the general equation of conic sections is usually not treated, though in some cases it is, as appears from the examination papers.

With the exception of a few schools only demonstrative geometry is taught; an intuitive course still meets with much opposition. Besides the geometry of Euclid often recent geometry is treated: Stewart's, Ceva's and Menelaus' theorems, the nine-point circle, the Simson line, perhaps (but certainly not frequently) the theorems of Desargues and Pascal (for the circle). Out-door measurements by the pupils with simple instruments, field work with the theodolite are unknown in Holland. The slide rule, too, is not in use in our schools; yet just in the last months computations with it have been recommended.

History and biography, rather neglected till now, begin to draw some more attention at present; there are however only few teachers, who are interested in this respect. A drawback certainly is that history of mathematics is not taught in the Universities of Holland, except—as far as I know—by Professor H. de Vries in the University of Amsterdam. Finally it must be added that in another respect Holland—and Europe in general indeed—has to learn from America, viz. in applying the principles of psychology to mathematical teaching. This science, that has, though comparatively young, already given rise to so many investigations in America[1], is little cultivated in Holland, at least with respect to mathematics.

V. *Textbooks and other accessories of teaching.* It is impossible to give even an abridged survey of mathematical textbooks in Holland, they are so numerous. The works of H. A. Derksen and G. L. N. H. de Laive[2] are still much used, as well as the more recent of A. van Thijn.[3] A great many textbooks were written by P. Wijdenes[4], partly in collaboration with others. Especially his works on algebra are modern and the function concept imbues some of them just as the Reform-movement demanded. Two works may separately be named: the Geometry of W. Rein-

[1] A good survey of these psychological methods may be found in the Report "The Reorganization of Mathematics in Secondary Education," pp. 279-428.
[2] Publishers: Thieme and Co. in Zutphen.
[3] Publisher: Wolters in Groningen.
[4] Publisher: Noordhoff in Groningen.

dersma[1], the only Dutch textbook in which an intuitive course precedes demonstrative geometry and remarkable too because of its historical annotations, and "Functies" of J. Droste and W. F. de Groot[1], a textbook of which the works of T. Percy Nunn[2] have been the prototype. Those, who should wish to become more acquainted with Dutch textbooks, are referred to the list, already mentioned, in Dr. Lietzmann's work on the teaching of mathematics.

All pupils must be provided of course with a ruler and compasses. The protractor is always a semi-circular one, never rectangular as often in England. The pupils draw their graphs on squared paper, while a squared blackboard is often used by the teacher. A fine set of 12 graphic representations (size 50 x 60 c.M.), fit for being hanged in the class-room, has been edited by P. Wijdenes (Publisher Noordhoff, Groningen).

VI. *How to be entitled to teach secondary mathematics in Holland.* There are different ways in Holland to obtain the right to teach mathematics in secondary schools. Many of the teachers have had a training at the University; those, who have successfully passed the examination for the doctorate and consequently the doctors in mathematics and physics are entitled to be a teacher in secondary schools. So, too, are engineers, having finished their studies at the Technical Academy at Delft. Besides there exist certificates of capacity, obtained by passing State-examinations, called KI and KV (these strange denominations are those of the law of 1863). This way is often followed by teachers of elementary schools. The examinations are decidedly difficult, at least nowadays. Those who have successfully passed KV (Higher Mathematics) may be appointed as a teacher in a H. B. S. with a five-year course.

While thus the scientific preparation of teachers in Holland is excellent and mathematical studies are indeed on a high level, the same cannot be said of a practical and theoretical preparation, which fails absolutely. The remark in Prof. Archibald's report[3]; that "in most cases the future teacher is transferred

[1] Published by Wolters, Groningen.

[2] T. Percy Nunn. The teaching of Algebra (including Trigonometry), and: Exercises in Algebra (including Trigonometry) I and II. Publishers: Longmans, Green and Co., London.

[3] R. C. Archibald. The training of teachers of Mathematics (U. S. Bureau of Education, Bulletin 1917, No. 27), p. 156.

directly from the university to the school, without ever having taught and without any knowledge of theoretical and practical pedagogy'' is still quite true, notwithstanding the intentions of Dr. de Visser, a former Minister of Education. A preparation as in Germany, where a seminary (training school) is attached to many secondary schools, is quite unknown here. University chairs for pedagogy are found in Amsterdam (Prof. Ph. Kohnstamm), in Utrecht (Prof. J. H. Gunning Wzn.) and in Leiden (Prof. R. Casimir). Prof. Casimir is assisted by W. Reindersma, who delivers in the University of Leiden lectures on the teaching of mathematics.

VII. *Associations of mathematics teachers in Holland. Periodicals.* The most prominent mathematical society, which represents the Netherlands in the scientific world, is the ''Wiskundig Genootschap'' in Amsterdam. This association, well-known in the United States, was founded as early as 1778 with the device ''Een onvermoeide arbeid komt alles te boven'' (i.e.: Diligence overcomes all difficulties). According to the Union List[1], edited in 1918 by Prof. D. E. Smith and Miss C. E. Seely, the publications of the society (''Nieuw Archief voor Wiskunde,'' ''Revue Semestrielle des Publications Mathematiques,''''Wiskundige Opgaven'') are found in many libraries in the United States. The Wiskundig Genootschap is in the first place a scientific society and does not, therefore, pay special attention to educational matters. This is perhaps the reason that among its members not so many teachers are found as might be expected.

Besides the publications of the Wiskundig Genootschap there are some other periodicals. First may be mentioned the ''Nieuw Tijdschrift voor Wiskunde,'' edited since 1913 by H. G. A. Verkaart and P. Wijdenes. It serves chiefly the interests of those, who prepare themselves by private tutors or by personal effort for the State-examinations, it is however important for every one, who has to teach mathematics. Another magazine, ''Christiaan Huygens,'' edited since 1922 by Prof. F. Schuh, is devoted to the higher parts of mathematics. In 1924 a Supplement, particularly devoted to educational questions (''Bijvoegsel van het

[1] D. E. Smith and C. E. Seely. Union List of Mathematical periodicals (U. S. Bureau of Education, Bulletin 1918, No. 9). The "Wiskundig Tijdschrift," that occurs in this list, ceased to exist in about 1920.

Nieuw Tijdschrift voor Wiskunde") was attached to the Nieuw Tijdschrift; it is edited by J. H. Schogt and P. Wijdenes. Publisher of these three periodicals is P. Noordhoff in Groningen.

In America there are, if I am well informed, three large associations of secondary school teachers of mathematics. Perhaps one might ask if such unions exist in Holland. There are indeed. Since 1921 there exists a committee of mathematics teachers under the auspices of the Genootschap van leeraren aan Nederlandsche Gymnasien" (i.e.: Society of gymnasium teachers in Holland). This committee (president Dr. D. P. A. Verrijp at Arnhem) has issued some reports and studies improvements of the mathematical teaching in the gymnasia. Recently (Dec., 1925) another union of mathematics teachers has been established, the "Vereeniging van leeraren in wiskunde, mechanica en cosmographie aan H. B. S." (i.e.: Union of H. B. S. teachers of mathematics, mechanics and cosmography), of which Dr. P. G. Tiddens in Utrecht is chairman. Probably the two groups will collaborate in future in many respects.

In conclusion I wish to thank all those, who, in giving me information, have assisted me in composing this article. They have greatly facilitated my task.

WORDS OF CLASSICAL DERIVATION IN COMMON MATHEMATICS VOCABULARY

By "Common Mathematics Vocabulary" is meant those words which are found in all three branches of high school mathematics, arithmetic, algebra, and geometry. The list of words used here is based upon the work in "The Technical Vocabularies of the Public School Subjects," by Mrs. Luella Cole Pressey of Ohio State University, as published by the Public School Publishing Co.

Accurate—L. accuratus, p. p. of ad + curare (to apply care to)

Accuracy—L. accuratio, -onis, n. cf. accurate, see note 1

Add—L. addere, ad + dare (to put to—not from dare, to give)

Addition—L. additio, -onis, n. (the act of adding to); see add

Altitude—L. altitudo, n. (height)

Angle—L. angulus, n. (corner), through Fr. angle

Antecedent—L. antecedens, -ntis, pr. p. of ante + cedere (to go before)

Approximate—L. approximatus, p. p. of ad + proximare (to draw near to)

Area—L. area, n. (an open space)

Arithmetic—L. arithmeica, n. from G. arithmetike adj. (arithmos) number; see note 2

Axes—L. pl. of axis

Axis—L. axis, n. (axle-tree, pivot)

Balance—Late L. bilanx, -bis + lanx (having two scales) through Fr. balance

Base—L. basis from G. basis (a stepping, that on which one steps)

Cancel—L. cancellare (to make lattice-wise, to cross out) through Fr. canceller

Centre—L. centrum, from G. kentron, (centre) through Fr. centre

Circle—L. circulus, dim. of circus (a little ring) through Fr. cercle

Circumference—Late L. circumferentia from circumferens, —ntis, pr. p. of circumferre (to bear around) through Fr. circonference.

Collect—L. collectus p. p., con + legere (to gather together); see note 3

Combinations—Late L. com + bini + atio, -onis, n. (a joining two by two); see note 3

Combine—Late L. com + bini + are (to join two by two)

Common—L. communis, adj. (common)

Compute—L. com + putare (to reckon together) through Fr. computer; see note 3

Consequent—L. consequens, -ntis, pr. p. of con + sequi (to follow closely); see note 3

Corresponding—Late L. correspondere, com + respondere (to answer to each other); see note 3

Cube—Late L. cubus from G. kubos (a die used in the game of dice)

Cubic—L. cubicus from G. kubikos, cf. cube; see note 2

Curve—L. curvus (bending, curved)

Cylinder—L. cylindrus from G. kulindros (a roller)

Decimal—Late L. decimalis from decem (ten)

Decrease—L. de + crescere (to grow less)

Degree—L. de + gradus (a step in descent) through Fr. degre

Denominator—L. denominatus p. p. of de + nominare (to give a name to)

Diagonal—L. diagonalis from G. diagonios, dia + gonia (from angle to angle)

Diameter—L. diametrus from G. diametros (the measure through)

Dimension—L. dimensio, -onis, n., from di + metiri (to measure out)

Direction—L. directio, -onis, n., from di + regere (to keep straight toward)

Distance—L. distantia, n., from distans, -ntis, pr. p. of dis + stare (to stand apart)

Divide—L. dividere (to divide, to force apart)

Division—L. divisio, -onis, n., from divisus, p. p. of dividere, see divide

Equal—L. aequalis, from aequus (level, even)

Equality—L. aequalitas, -atis, n. see equal.

Equation—L. aequatio, -onis, n. (a making equal)

Equilateral—Late L. aequilateralis, aequs + latus, -eris (having equal sides)

Equivalent—Late L. aequivalens, -ntis, pr. p. of aequus + valere (to have equal power)

Example—L. exemplum, n. (something taken out as a sample)

Express—L. expressus, p. p. of exprimere, ex + premere (to press out, portray)

Extract—L. extractus, p. p., of ex + trahere (to draw out)

Extremes—L. extremus, adj. (outermost, most remote)

Factoring—L. factor, n. (doer or maker)

Figure—L. figura, n. (shape or form)

Formula—L. formula, n. (small pattern)

Fraction—L. fractio, -onis, n. (a breaking) cf. frangere, fractus

Geometry—G. geometria, n. (land-measure) from G. ge, land + metron, (measure)

Graph—G. graphe, n. (a writing)

Graphical—G. graphikos, adj. (pertaining to writing or drawing) + al; see note. 4

Horizontal—G. horizon, n. (the bounding circle) + al; see note 4

Hypotenuse—G. hypoteineusa, p. p. of hypoteinein (to stretch under)

Increase—L. in + crescere, (to grow)

Integer—L. integer, adj. (untouched, whole)

Integral—Late L. integralis, adj., see integer

Interpretation—L. interpretatio, -onis, n. (explanation)

Intersect—L. intersectus, p. p. of inter + secare (to separate, cut asunder)

Invert—L. in + vertere, (to turn upside down)

Isosceles—G. isoskeles, adj. (with equal legs)

Linear—L. linearis, (having to do with a line), from linea (line)

Mathematics—Late L. mathematica, from G. mathematika, neut. pl. adj. mathematikos, from n. mathema (something learned), from v. manthano, (to learn)

Measure—L. mensure, n., from mensus, p. p. of metiri (to measure) through Fr. mesure

Measurement—L. mensura + ment (a suffix forming substantives from verbs, cf. L. -mentum); see note 5

Method—Late L. methodus from G. methodos (pursuit of knowledge), meta (with) + hodos (way)

Minus—L. minus, adj. neut. of minor (less)

Negative—Late L. negativus, adj. from negatus, p. p. of negare (to deny)

Notation—L. notatio, -onis, n. from notare (to note)

Number—L. numerus (number) through O.Fr. nombre

Numeral—Late L. numeralis, adj., from numerus (number)

Numerator—Late L. numerator, n., from numerare (to number)

Numerical—Modern L. numericus + al, from L. numerus (number)

Oblong—L. ob + longus (somewhat long, longish); see note 6

Operation—L. operatio, -onis, n., from operari (to work), from n. opus, oper-is (work)

Order—L. ordo, -inis, n. (a row, series) through O.Fr. ordre

Origin—L. origo, -inis, n., (rise) from oriri (to arise)

Parallel—L. parallelus, from G. parallelos, para (beside) + allelos (one another) through Fr. parallele

Partial—Late L. partialis; cf. pars, part-is

Per Cent—L. prep. per (through, by) + Fr. cent, or L. centum (hundred)

Perimeter—L. perimetros, from G. perimetros (circumference), from peri (around) + metron (measure)

Perpendicular—L. perpendicularis, from perpendiculum (plummet, plumb line)

Pi—G., Sixteenth letter of the Greek alphabet, used as a symbol

Plus—L. plus, adj. (more)

Point—L. punctum, n, (a small hole), through F. point

Polygon—G. polugonos, adj. (many angles)

Positive—L. positivus, adj. (settled by agreement) from positus, p. p. of ponere (place)

Principal—L. principalis, adj., (first, original, important), cf. princeps

Problem—G. problema, n. (something thrown or put forward)

Proportion—L. proportio, -onis, n. (comparative relation, likeness)

Proportional—proportion + al; see note 4

Pyramid—G. puramis, -idos, n. (a flat-sided object)

Quadrilateral—Late L. quadri + laterus, adj. (four-sided)

Quantity—L. quantitas, -atis, from quantus, (how much) through O. Fr. Quantite

Quotient—L. quotiens, adv. (how often?)

Radius—L. radius, n. (ray, spoke)

Ratio—L. ratio, -onis n. (a reckoning, account, calculation) cf. reor, ratus

Rectangle—L. rectus (straight, upright) + angulus (corner, angle)

Reduce—L. re + ducere (to lead or bring back)

Remainder—L. re + manere (to stay back), through O.Fr. remaindre

Represent—L. re + praesentare (to bring before one, to show)

Required—L. requirere, re + quaerere (to look after, seek)

Respectively—L. re + spicere (to look back upon)

Sign—L. signum, n. (mark or token)

Similar—L. similis, adj. (like), through Fr. similaire

Simple—L. simplex, simplicis, adj. (uncompounded, unmixed)

Solid—L. solidum, n. (a dense, compact substance)

Solution—L. solutio, -onis, n. (a loosing or release), cf. solve

Solve—L. solvere (to loose, release)

Sphere—G. sphera, n. from G. sphaira (ball)

Square—Late L. exquadrare (to make square), from quadrus (four-cornered) from root of quattuor (four), through Fr. esquare

Substitute—L. substitutus, p. p. of substituere, sub + statuere (to set up in place of)

Subtract—L. subtractus, p. p. of sub + trahere (to drag away)

Successive—L. successivus, adj., formed from participial stem of succedere, sub + cedere (to come close to)

Sum—L. summa. n. (the complete amount)

Supplement—L. supplementum, n. (a supply, reinforcement) from sub + plere (to fill); see note 5

Symbol—G. symbolon, n. (mark, token, ticket)

Terms—L. terminus, n. (limit, boundary-mark)

Transpose—L. trans (across) + Late L. pausare through Fr. poser, confused with ponere (place)

Triangle—L. triangulum, n., tres + angulus, (a figure having three angles)
Unit—L. unitas, n. (oneness)
Value—L. valere (to be strong, to be worth), through Fr. valu, p. p. of valoir.
Vertex—L. vertex, n. (top or summit), cf. vertere (to turn)
Vertical—vertex + al; see note 4
**Volume*—L. volumen, n. (a roll of writing), cf. volvere (to roll)

NOTES

1. -acy, suffix from Latin -acia, -atia, p. p. forms in-atus, Greek -ateia, -atia.

2. L. -icus, G. -ikos, adj. ending meaning "of" or "belonging to."

3. Cum or com—in compounds com, con, col, cor.

4. -al, from Latin -alis, adjectival ending indicating connection or relation.

5. -mentum, Latin suffix expressing result or means of verbal action from which are formed English nouns in -ment, either direct or through the French.

6. Prefix ob, from latin preposition ob, meaning in the direction of.

LIST OF ABBREVIATIONS USED

adj.—adjective	n.—noun
cf.—compare	neut.—neuter
dim.—diminutive	p. p. past participle
Fr.—French	pl.—plural
G.—Greek	pr. p.—present participle
L.—Latin	prep.—preposition
Late L.—Late Latin	v.—verb

* There are thirteen additional words belonging to the "common mathematics vocabulary" which are not at all or only in part of classical origin.

A COURSE IN SOLID GEOMETRY

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The Problem of Solid Geometry. No subject in high school mathematics receives less constructive consideration than does solid geometry. No subject is more poorly taught nor more useless. To many pupils it is most uninteresting and to many teachers it is mere drudgery. There are more expressions of dissatisfaction with classroom procedure and results obtained in this subject than in any other. If presented immediately following plane geometry, as is usually the case, it is irksome and aimless both to pupils and teacher. If taught later on in the course to pupils who have had advanced algebra or trigonometry or both, its apparent value is less obvious unless the teacher liberates the class from the type of work given in the ordinary text book.

Text books in solid geometry have changed little in content, arrangement, or material. This condition is due, no doubt, to the fact that thought and energy of authors are focused upon plane geometry primarily; and the solid geometry which follows as a necessity is thrown together hurriedly. Teachers, as a rule, follow the text closely and use little originality. This is perfectly natural, for their duties are so numerous that time does not permit them to prepare any outside material. Hence, the text must supply the work as it should be. It is so easy to assign pages from a book and to "hear recitations." The task of modernizing the work in this subject, therefore, has not yet been undertaken in earnest.

The necessity for a reorganization of its subject-matter, for a restatement of its objectives, and for a transformation in the method of attack and procedure is evident. The necessity for a revision of solid geometry is obvious for at least two reasons. First, the usual text book method does not conform to present-day requirements of educational theory and practice nor to the demands of modern social and civic life. Second, those pupils who must take the subject in order to pursue certain technical

courses in college are practically the only ones signing up for the course. Since many courses in mathematics are being made elective for graduation, pupils naturally drift in herds to those courses which appeal to their interest and which seem to them most useful. It frequently happens also that pupils choose one subject rather than another on a basis far different from that of an idea of its value to them as students or citizens.

The Purpose of This Article is to advance a plan for the reorganization of the subject-matter of solid geometry, to suggest worthy objectives and results to be attained, and to propose a more effective method of procedure. The work herein outlined has been used by the author for several years with some degree of success. The idea of this paper is to open the subject up for discussion, rather than to set forth a finished product. We are anxious to obtain a reaction from every reader.

Pre-requisites. Our course in solid geometry is given the last half of the fourth year of high school mathematics. Before entering this subject the pupil has passed in the following: first year algebra, one year of plane geometry, second year algebra, and one-half year of plane trigonometry.

There are two main reasons why we choose to place solid geometry the last half of the senior year. Since the California State law provides that three year's work in a subject constitutes three units of a major towards graduation, we wish pupils to become more efficient in algebra and to know something about trigonometry if they drop mathematics after making a major of it. By placing solid geometry in the last half of the senior year, we can employ algebra, trigonometry, and advanced mathematical ability developed in pupils, and the maturity of thought which a year or two in age means, to make this course just what we intend it to be.

Objectives of the Course. Our course serves as a review in actual practice, not only of plane geometry but of algebra and trignonometry as well, since these subjects are used extensively. The process of algebra previously taught are made real, useful, and vital. Pupils learn to use mathematical tools for definite purposes. We open up the subject of space and develop the

conceptions of space relations necessary in advanced work in mathematics. Mathematical conceptions and operations are made definite, objective, and quantitative, capable of being measured, constructed, and computed. We try to eliminate the theorical and purely speculative aspects of the subject in so far as possible. Formulae for all common geometrical solids are developed and problems of practical application are introduced to make the course valuable to any man or woman.

Subject-matter. The material usually given in a text on solid geometry is presented. Theorems, problems of construction, and computations involving surfaces and volumes of solids are studied in a more advanced manner than that usually employed when this course immediately follows plane geometry. The usual demonstrations of almost self-evident propositions are minimized and the development of algebraic and trigonometric formulae is emphasized. This course also contains an introduction of problems leading to the idea of functions of variables, graphical representation of functions, questions of maximum and minimum, and the calculus. In dealing with spheres many points of theory ordinarily found in texts are neglected, if necessary, for the purpose of developing a few formulae in spherical trigonometry; and selected problems involving geodetic surveying and astronomy are introduced in application of these formulae.

Method of Attack. This is our fundamental point of departure from the usual work in this subject. Emphasis is given to problems of construction and of computation. Theorems are proved when they become a necessity to establish the correctness of a construction, a computation, or of an observed situation. In almost every construction there arises questions as to the correctness of results. Thus arises the necessity for a proof of an apparent general truth. This general truth is stated as a theorem, demonstrated, and catalogued among others. Few theorems are presented formally.

The details of our method will appear in the statement of subject-matter in outline which follows. An illustration at this point, however, will be of value. Attention is called in our discussion to the plan used by a carpenter in scratching lines for sawing a right section from a prismatic beam. A point P is de-

termined on one edge of the prismatic space represented by the beam. With the use of a "square" a line is drawn in an adjacent face plane perpendicular to that edge at point P. In the other adjacent face plane a similar line is drawn. These two lines guide the saw in the plane of sawing. Three questions arise: first, is the plane of sawing perpendicular to that edge of the prismatic space? Second, if the plane of sawing is perpendicular to this one edge is it also perpendicular to all edges? Third, does the plane of sawing produce a right section? Hence, there comes the necessity for stating two theorems, one of which has been previously demonstrated and the other requires proof. 1. A plane is perpendicular to a line if it is determined by two lines each of which is perpendicular to the given line at a given point. 2. A plane perpendicular to one of a system of parallel lines in space is perpendicular to each. The definition of a right section of prismatic space answers the third question.

Each item in the subject of solid geometry is introduced in a similar manner. After a brief discussion of space, solids, and planes in space, we discover that we must study points, lines, surfaces, and solids in what is called space. The first natural problem which arises in our minds is concerning the definite location of these geometric figures in space. We remember how we located points and lines in a plane by a system of rectangular co-ordinates. We carry over this idea to space, select three planes perpendicular to each other, and employ the scheme of space co-ordinates. Hence, we begin at once locating points, lines, planes, and solids in space by the co-ordinate system.

Method of Study. We attempt to study solid geometry scientifically and objectively as well as theoretically. We might even venture to say that a modified laboratory plan is used. We can see no real reason why the principles and laws involved in space relations cannot be studied objectively and in a scientific manner in what might be called a laboratory just as well as the principles of physics. We cannot understand, for example, the logic in making Boyle's Law objective, experimental and capable of computation, and at the same time treating a law in solid geometry purely theoretically, assisted only by a free hand sketch. We may set up an apparatus to demonstrate the construction of the

perpendicular from a point in space to a given plane, just as easily and as fruitfully as we can set up an apparatus in the physics laboratory to demonstrate the law of the incline plane. We know from experience that this plane of studying solid geometry produces wonderful results, lasting impressions, keen interest and enthusiasm, and an appreciation of its usesfulness.

Recitations. Parrot-like recitations are unknown. Such recitations are of no service to teacher or to pupil. Drawings and demonstrations are not copied on the board from the text, neither does a pupil stand facing the board and read off the copy. Instead, a pupil stands before the class and with the assistance of other pupils uses sticks for lines, the floor, table top, or cardboard for planes, sets up the apparatus and discusses the construction and demonstration as he proceeds. He gives what might be called an illustrative lecture.

Home Work and Preparation for Class. Drawings are made to show the work illustrated and discussed in class. Complete discussions and demonstrations are frequently written out in full on paper near the drawing on a "plate." Many pupils prepare solid geometry plates at home, in the drawing room, or in the study hall. Others who have had mechanical drawing produce ink plates, tracings and blue prints. Exercises and problems are prepared outside of class.

Outline of Material. The following outline of subject-matter, of necessity, is abridged; but the reader may be able to discover the fundamental ideas in our course.

I. Fundamental Concepts and Definitions

1. *Space.* A discussion of space concludes with some idea of what space is and of its extent. In connection with this discussion the subjects of astronomy, time, eternity, and the fourth dimension are introduced. A solid and solid geometry are defined.

2. *Space Co-ordinates.* The idea of locating geometric figures in space is as easily comprehended as the plan of locating points on a plane. Three planes perpendicular to each other are selected as planes of reference by which points, lines, planes, and even

solids are located by the system of co-ordinates. A corner of the room illustrates the three planes of reference. A box or carton with the top, one end and one side removed may also be used to demonstrate the idea.

3. *Locating Points, Lines and Planes in Space.* The three perpendicular planes, 1, 2, and 3, (Fig. 1) intersect at point *O* and by pairs in lines *OX*, *OY*, and *OZ*. These are the three planes selected for reference.

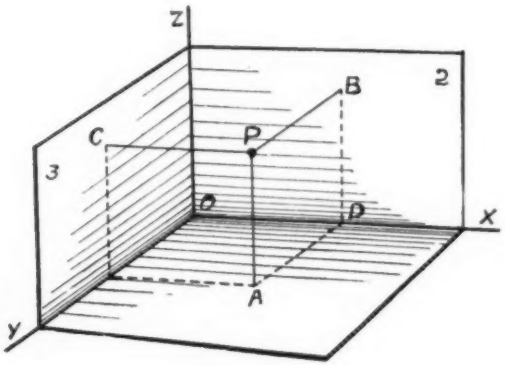

Figure 1

a. *Locating a Point.* A point in space is located by giving its co-ordinate distances from these three planes. The conception can be made simple and concrete by using the corner of the room, or the carton, or by drawing it on paper. Figure 1 is a drawing to show the point in space about as the eye would see a ball if it were fixed in position in the carton. *PA*, *PB*, and *PC* represent wires holding the ball, P, given distances from the planes. The length of these three wires are the three co-ordinates of this point. To locate a point in space, therefore, three numbers, x, y, and z, (co-ordinates) are given. $OD = x$, $DA = y$, and $AP = z$. This device fixed up in a box or carton gives the pupil an excellent basis for drawing space relations on plane paper. The reader may project his thought at this point and realize the many possibilities involved in this idea of locating points in space. He may also know that this conception is the fundamental principle underlying projective geometry and geometric and mechanical drawing.

b. *Locating Lines.* A line in space may be located by fixing two points in space. Figure 2 is a drawing to illustrate line AB,

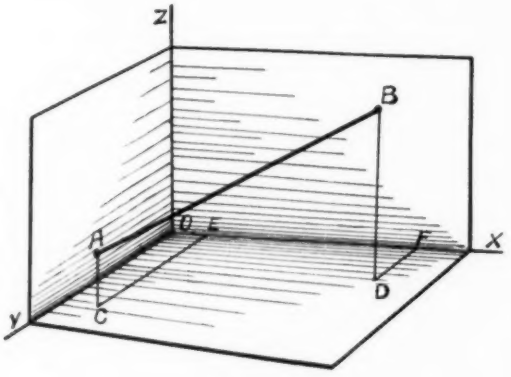

Figure 2

determined by the two points, A and B, which are located in space as shown in Figure 1. Axiom I. *Two points in space determine one line.*

c. *Locating a Plane.* A plane in space may be located by fixing three non-colinear points. To make this fact objective, let three balls be fixed in a carton and a cardboard or glass be placed

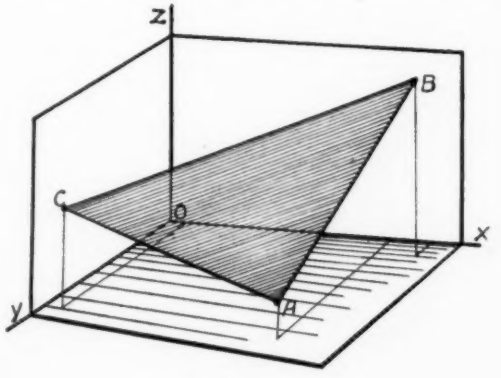

Figure 3

on the balls so that it may rest firmly on them. The cardboard or glass is fixed in space. Figure 3 is a drawing to represent a plane determined by the three fixed points, A, B, and C. Axiom

II. *Three non-colinear points determine one plane.* Three corollaries to this axiom may also be illustrated objectively in a similar manner. 1. A line and a point not in that line determine a plane. 2. Two intersecting lines determine a plane. 3. Two parallel lines determine a plane.

Problems involving the location of points, lines and planes are given in abundance. Balls, sticks, and cardboard held by pupils in the corner of the room or fixed in the carton represent to positions of points, lines and planes given in the problems. Drawings for each are made. Problems involving the length of lines and the inclination of lines to a plane in terms of the co-ordinates of their ends may be given as supplementary work if the strength of the class justifies.

4. *Relations among Lines and Planes in Space.* (a) A straight line may pierce a plane; it may be perpendicular to or oblique to a plane; or it may be parallel to a plane. (b) Relations among planes in space. A plane may rotate on one of its lines, forming intersecting planes, and thus generating a space or dihedral angle. The dihedral angle is measured by a certain plane angle. Two intersecting planes may be oblique to each other or perpendicular to each other, determined by the value of the measuring angle of the dihedral formed. Two planes may also be parallel.

5. *Lines and Planes in Space with Relation to the Earth.* A vertical or plumb line is determined by a plumb line and bob. A level or horrizontal line is determined by a spirit level. A horrizontal plane is determined by two intersecting level lines. The carpenter's level, the engineer's transit, the architect's level, and other instruments used in determining these lines and planes are described and shown in this connection if these instruments can be obtained. A vertical plane is determined by a plumb line. Problems and questions involving architectural construction and engineering are proposed for discussion.

II. *Problems of Construction.* Real work in geometry begins with the introduction of many problems of construction. We will present here the first problem and briefly outline the work in connection with it.

Problem 1. *To erect a perpendicular to a given plane at a given point in the plane.*

Given: Plane (1) and point P in that plane. (See Figure 4.)
To erect a perpendicular to plane (1) at point P.

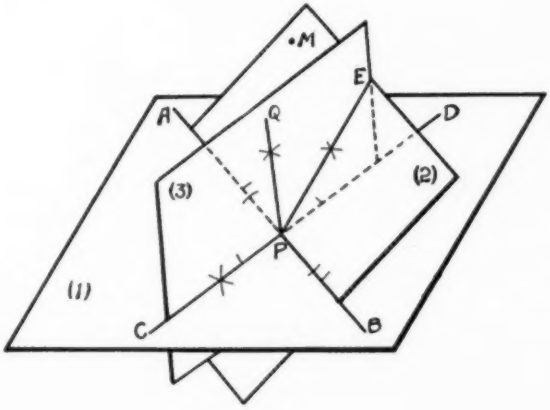

Figure 4

Construction:

(1) Draw line AB in plane (1) and passing through point P.
(2) Select any point M outside plane (1).
(3) Point M and line AB determine plane (2).
(4) In plane (2) erect PE perpendicular to AB at point P.
(5) In plane (1) erect CD perpendicular to AB at point P.
(6) Lines CD and PE determine plane (3).
(7) In plane (3) erect PQ perpendicular to CD at point P.
(8) Line PQ is the required line.

The construction given above is made objective by having pupils stand before the class, choose the floor as the given plane, and place point P on the floor with crayon. A line through point P is drawn on the floor, a ball is held for point M, cardboards or glass are used for planes (2) and (3), and a stick is held in position for line PQ. In this manner the pupil goes through the details of the construction. After this is clearly demonstrated there is little difficulty in reproducing the construction on plane paper.

Now, in order to prove that this construction is correct and that PQ is the required line, two questions arise and must be answered. First, is a line perpendicular to a plane if it is perpendicular to each of two lines in that plane and intersecting in its piercing point? Second, if a plane is perpendicular to a line in a plane, is it perpendicular to that plane? Hence, it is obvious now that there must be a definition of a line perpendicular to a plane and the statement and proof of two theorems.

Other problems of construction are:

2. To construct a perpendicular to a given plane from a given external point.

5. To construct a plane perpendicular to a given plane and also containing two given points.

8. To construct the measuring angle of a dihedral angle.

10. To construct the planes which bisect a given dihedral angle.

12. To construct a plane parallel to a given plane and passing through a given external point.

15. To construct a plane parallel to two given lines in space and also containing a given point.

16. To construct the common perpendicular to two given planes in space.

17. To construct the angle a line makes with a given plane.

18. To construct the shortest distance from a point to a given plane and thence to another given point on the same side of the plane.

In connection with the study of these constructions there arises the necessity for the statement and proof of many propositions and numerous corollaries. The question of locus also comes up for consideration. Many problems of application and concrete illustrations are introduced.

III. *Projections.* Under this heading comes the discussion of the projection of a point and of a line on a plane. In presenting the idea of a projecting plane, there is the necessity for proving the theorem: Lines perpendicular to the same plane are parallel. The length of the projection of a line on a plane involves two problems: (1) when the length of the line and the angle it makes with the plane are given; and (2) when the co-ordinates of its ends are given.

Many problems under this heading involving trigonometry and algebra are introduced. The discussion of the use of projections as used in mechanical and architectural drafting depends upon the limits of the ability of pupils.

IV. *Prismatic Solids.* Definite notations concerning the following are obtained: prismatic space, sections of prismatic space, right and oblique sections, prism, right prisms, parallelopipeds, the cube and the box prism; and the volume and total surface of the cube and the box prism. Two sets of problems follow: one group containing problems for numerical computations; the other containing algebraic problems. A treatment of the surface and volume of prisms in general is followed by two sets of problems, arithmetic and algebraic.

V. *Polyhedral Angles.* Definitions of and drawings for the following are made: polyhedral angle, pyramidal space, sections of pyramidal space, pyramids, and classes of pyramids.

(a) Regular pyramids. Formulae for lateral and total surface and for the volume of regular pyramids are developed. Two sets of problems, as usual, follow.

(b) Regular pyramids are made by cutting patterns from cardboard or tin. We solve for the radius R and the central angle X^0 of the sector that will build a regular pyramid of required altitude and base dimensions. The converse of this problem is also given.

(c) Problems involving dihedral angles and slopes of faces of regular pyramids. The "Great Pyramid" of Egypt is one illustration.

VII. *Frustum of a Pyramid.* How a frustum is obtained from a pyramid. Development of formulae for total surface and for volume. The construction of a frustum, such as a lamp shade, by cutting a pattern from cardboard or tin. Two sets of problems. The Washington Monument at Washington, D. C., is one numerical problem.

VIII. *Cylinders.* Definition of cylindrical surface, elements of, cylindrical space, sections of, and cylinder. Development of formulae for total surface and for volume of cylinders. Two sets of problems. Prisms inscribed and circumscribed about cyl-

inders. The question of limits. Many algebraic problems are given. The following are samples: Find the per cent waste in turning a right prism with a square base and given altitude into the largest cylinder. Find the per cent waste in sawing four equal slabs from a right circular cylinder to form the largest square beam. Development of formulae for total surface and volume of the washer or pipe. Two sets of problems.

IX. *Cones and Frustums of Cones.* The cone and frustum of the cone are developed in a manner similar to the treatment of the pyramid and frustum of the pyramid. Methods of making right circular cones and frustums by cutting patterns. Problems involving arithmetical algebraic, and trigonometric computations are given.

X. *Surfaces of Revolution.* Ten problems dealing with surfaces generated by revolving a line in a plane on another line in that plane as an axis are either worked out in class or assigned to pupils for special reports. Algebra and trigonometry are used in these problems. The following theorem is proved: The surface generated by a line revolving about another coplaner line equals the projection of the line on the axis times the circumference of a circle whose radius is the length of the perpendicular bisector of the line terminated by the axis. A corollary to this theorem gives this surface in terms of the length of the line and the angle formed by the line and the axis.

Twenty-six problems involving the surface and volume generated by revolving certain polygons, such as squares, rectangles, and triangles, about a coplaner axis, are either solved out in class or assigned to pupils for special reports.

XI. *The Sphere.* The method of the treatment of the sphere has been explained previously in this article.

XII. *Problems Leading to the Calculus.* This work is introduced by presenting and solving in detail the following problem. Given a rectangular piece of tin. Equal squares are cut from each corner and the sides are bent up to form a box.* (a) Find the volume of the box as a function of x, one side of the square

*See "A Laboratory Plane Geometry," by Austin, page 296, for a similar method of treatment. See also page 301. Scott, Foresman Company, Chicago.

cut out. (b) Plot the volume as a function x. (c) Find the value of x such that the volume of the box shall be a maximum.

The first and second parts of this problem offer very little difficulty. In solving the third part of the question, however, the ideas of tangents to a curve, of the slope of tangents, of maximum and minimum points on the curve, of the value of the slope of the tangent drawn at the maximum or minimum points of the curve, and the derivitive of a function must be taught. Pupils grasp a simple demonstration of the meaning, application, and computation of the derivitive very readily. They quickly become efficient in obtaining the derivitive of simple algebraic functions.

"The calculus of the algebraic polynomial is so simple that a boy or girl who is capable of grasping the idea of limits, of slope, or of velocity, may in a brief time gain an outlook upon the field of mechanics and other exact sciences, and acquire a fair degree of facility in using one of the most powerful tools of mathematics, together with the capacity for solving a number of interesting problems."* Such work as this when presented to pupils who have a good foundation in algebra, geometry, and trigonometry is of far greater value to the pupil either as a prospective college student or to one who never attends a university, than the usual deadening grind over the proof of obvious theorems in the ordinary text in solid geometry.

A list of well-graded problems follow the detailed solution of the above problem. Two examples of such problems are:

1. Post-office regulations prescribe that the combined length and girth of a parcel must not exceed six feet. Find the maximum volume of a parcel in the shape of a right prism with the ends square.

2. Circular tin cans closed at both ends must contain a given volume. Find the dimensions of the can so that it can be made out of a minimum amount of tin.

Let us repeat, in conclusion, that the foregoing discussion and outline are mere suggestions. Mimeograph copies of the work as described herein and blue prints of constructions have been prepared and are now in use. What we desire, above everything else, is the reaction of every reader.

*See "The Reorganization of Mathematics in Secondary Education." Report by National Committee, pages 4-40-42.

CHECKING UP THE PROPHETS

By GEORGE W. EVANS
5219 Caroline Boulevard, Houston, Texas

The Yearbook of the National Council of Teachers of Mathematics, published last February, contains much of permanent importance, and in particular two papers by men of foremost authority and influence. One is by Professor E. H. Moore, his presidential address to the American Mathematical Society in 1902, a plan for progress and a hopeful forecast at the beginning of the new century; the other a cheerful summary of progress since that time and some indication of present tendencies by David Eugene Smith.

Professor Moore made several definite suggestions, based in part upon certain reforms then being begun and since well advanced in England. Among these suggestions were the following:

(1) That our ultimate aim should be to develop a coherent course in algebra, geometry and physics, "comparable in strength and closeness of structure with the four years course in Latin." Some parts of astronomy and physiography would be included under physics. This unification "is the fundamental problem."

(2) That in geometry a larger body of basal principles should be assumed, leaving most of the philosophic discussion to later years.

(3) That the Society should form co-operative relations with secondary schools, and give continuous attention to the question of improvement of education in mathematics, in institutions of all grades.

Dr. Smith's paper concerns itself mainly with the details of change in the content of arithmetic, algebra and geometry, as separate subjects, and presents with enthusiastic approval most of the reforms recently proposed by the National Committee on Mathematical Requirements and adopted by the College Entrance Board. It is interesting to compare the prophet and the historian.

First the prophet: an eminent mathematician, practised in teaching, familiar with the results of secondary school work,

speaking to a representative body of learned men, and urging important changes in organization and in pedagogy; timely changes, not devised by himself, though he might well have devised them, but only put by him into formal shape as the well informed opinion of many thoughtful men of the time.

Then the historian: a quarter of a century later, another eminent scholar brings forward plans for further progress, surveying the period intervening. He also is practised in teaching, and familiar with the results of secondary school work; he was a member of that earlier audience; he is now, and was even at that time, a historian pre-eminent for research in this special field; his opinion is consequently based on very full knowledge, and is of great weight. He sees now apparently little or no result from the plans then so auspiciously made, and in listing the influences that have in that period borne fruit, sees no reason to mention those plans or their author.

It may well be that Dr. Smith, since he knew that Professor Moore's address would be printed beside his own paper, thought the address would speak for itself, and to a certain extent it does. Fuller co-operation between colleges and secondary schools came as an early consequence of it, and the considerable decrease in formality of instruction is undoubtedly also to be attributed to it. It is true, besides, that the character of high school instruction is perforce changed, and some surrender of preference for mathematics has been in wide regions politically unavoidable. For that reason a four years course of mathematics and physics in thoroughly organized combination must now, for public secondary schools, be stored away among the Utopias. There remain other pedagogical suggestions of great value, and of these I shall speak of only two. One especially concerns mathematics, and the other is of general import.

The first of these is that in computation (or in geometrical construction) only a definite amount or percentage of error should be permitted. By this the pupil will be led to the general notion of progressive approximation to a perfectly definite result, and thus eventually in a practical way to the fundamental notions of limits and of irrational numbers. This suggestion has been carried out generally in England, and in this country

sporadically. It has been embodied in the Report of the National Committee. The form it has taken is to confine the number of decimal places used in computation to those warranted by the data, and for that purpose to change the rule of multiplication so as to begin at the left of the multiplier. It is thus not an "inert idea," as Carson would say, but of immediate and obvious advantage in practical computation; at the same time it has a deferred value, leading to the later use of these practical approximations and of their theoretical extensions in a "dense" series which serves to determine the irrational lines, ratios, areas, and volumes of elementary geometry. The very least that can be done by the secondary school is to lay well this foundation; it is not unreasonable to hope that something more than the foundation can be built up.

The other suggestion refers to the laboratory method of instruction. A laboratory is not necessarily a factory-built set of tables and lockers and shelves, with standing room only; it does not absolutely require test-tubes or micrometers or electric batteries. It does require that pupils should be getting things done and that teachers should be at hand to lay out the work, to see that it can be done, and to furnish the plans and the organization that inexperience cannot arrive at. Students can be handled individually, in small or large groups, or in committee of the whole. They can be organized to help each other, to pass down the achievements of the eager van, to discuss the perplexities of the cautious and the slow. A word from the teacher will do for some, will even be unnecessary for a few, can be expanded into encouraging talks for individuals beginning to sprawl. In very large divisions, and for occasional relief, assistants comparatively inexperienced will be of effective service. It is a means to secure much individual teaching without great expense. Those who object that such methods are appropriate only to classes in experimental science should know about their application to the teaching of history, for example by Sanderson, late headmaster of Oundle in England.[1]

There is still reason to hope that these two proposals will be carried out to the advantage of education; the second, referring

[1]History: the Quarterly Journal of the Historical Association. London, Macmillan. October, 1925, pp. 242ff.

to "laboratory" and co-operative methods, seems very promising for all subjects of instruction. As Professor Moore stated it, it was involved with his other proposal for the unification of mathematics, including topics in physics; the whole was to be without "water-tight compartments."

Physics is still tightly shut off, but between the compartments of mathematics the bulkheads are beginning to leak. Myers and Breslich of Chicago have made some openings, and everywhere the separation between high school algebra and elementary school arithmetic is becoming less and less effective; but around geometry, says Dr. Smith, the barriers must be maintained, or geometry will be so diluted as to be hardly worth saving. It is going to be hard for some of us to follow him in that.

The chief purpose of demonstrative geometry, says Dr. Smith, is to show the application of logic to mathematical statements; the purpose is not mensuration. The mensuration of the circle, *for this reason*,[2] has no sanction in this subject. Numerical trigonometry naturally blends with algebra, and *makes no use of the demonstrations of geometry in its work*. Demonstrative geometry can use the algebraic equation in its proofs, *although it can get along about as well without it*. Finally, if demonstrative geometry is not considered "largely by itself," it will remain, if at all, "as a feeble memory of the world's effort to show how truth is logically established in the mathematical sciences."

I wish he had said that demonstrative geometry must use and always has used the algebraic equation in its proofs, whether in the clumsy notation of old times or in our more convenient forms; that numerical trigonometry requires geometry for its fundamental theorems and itself contributes to the understanding of geometry; that geometry not only had its beginning in mensuration, but was by mensuration led to its most brilliant and famous logical achievement; and that if geometry is considered "largely by itself," excluding the things that Dr. Smith excludes, it will be more feeble than any memory.

[2]Italics all mine.

CLASSROOM METHODS IN TEACHING ALGEBRA

By JOSEPH A. NYBERG
Hyde Park High School, Chicago

The object this afternoon is to acquaint you with some of the classroom conditions and methods in a large public high school, and to show how the size of the classes and the fact that it is public rather than a private school influences our classroom methods.

I wish first of all to read you a part of a letter that is mailed each semester to the parents. I omit the section dealing with fraternities and sororities, choice of studies, and scholarship reports. The other part reads:

"If a pupil of ordinary ability does today's work today, he will have no difficulty in securing credit for high school work. Irregularity in work and in attendance almost always accompanies failure. Since on entering high school a pupil begins work entirely new to him, e. g., algebra and a foreign language, the great danger lies in failure thoroughly to master fundamentals; there must be sufficient practise to give facility in their use. These fundamentals, the operations in algebra and the declensions in a foreign language, should be at a pupil's finger tips, the same as the multiplication table.

There is not time in school for the necessary practise and drill. This is most emphatically true now, because we are running on a double schedule.[1] In addition to practise and drill, the pupil must train himself to get information from the printed page. The ability to get such information for and by himself is of greater importance than the information obtained. This again emphasizes the necessity of doing today's work today, when the

Presented at the University of Chicago Conference with Secondary School, May, 1925, and at the Iowa State Teachers Association Meeting, Des Moines, Iowa, November, 1925.

[1] Years ago it was customary to begin classes at 8:45 and end at 3:15. During the last forty minutes of the day both teacher and pupil were free. The pupils could then see the teachers for special help. Now classes extend from 8 to 4:30 and there is no period when teachers have an opportunity to meet a backward pupil. The pupil's program is also so arranged that he is at school chiefly for recitations and has a minimum of study periods in order that the rooms may be available for other classes to meet.

teacher's specific suggestions with reference to it are fresh. We estimate that the average pupil should spend from forty-five to sixty minutes on each lesson requiring outside preparation."

Notice particularly two of the sentences in this letter: "There is not time in school for the necessary practise and drill." and "The pupil must train himself to get information from the printed page." These sentences explain why our methods are different from a school in which most of the pupil's work is done in school under the supervision of the teacher.

The average size of the classes in algebra at Hyde Park last semester was 36 1/3. The total number of pupils for each teacher of mathematics was 150, 155, 168, 177, 178, 182, 186, and 186. The corresponding figures for the previous semester runs from 158 to 195.

My own method for dealing with this situation is as follows: I divide the class period of forty minutes into three parts, not necessarily equal in length. The first part of the period is devoted to diagnosis, the second part to drill, and the third part to the preparation for the homework or the next day's assignment.

Let us look first at the third part of the period when the teacher is preparing the class for the next day's work. I believe that the traditional method has been about as follows: The teacher steps to the blackboard, and having got the attention of most of the pupils, proceeds to work a problem, explaining each step as he goes along and answering any questions that may arise. Next the pupils try a similar problem, and then the teacher assigns 10 to 15 similar problems to be done at home. Much too often the assignment and preparation for the next day's work consists in having the teacher say, just as the bell rings, "Do exercises 11 to 20 for tomorrow."

If our object is that of teaching the pupil how to work one particular type of problem, then undoubtedly the easiest way for the pupil is to watch the teacher work one of the problems. If, however, we wish to teach something more than algebra, if we agree with the statement that "the pupil must be taught how to get information from the printed page" and should be taught self-reliance (that is, independence of the teacher), then this method is a very poor one. I prefer a method based on the

assumption that textbooks should be something else than a collection of exercises, that textbooks are meant to be read not merely by the teacher but also by the pupil.

The teacher should see to it that the explanation in the book is read by the pupil, and that the pupil dig out the explanation from the book. Also, this digging should be done *in class* and not at home. Hence the explanations should be read in class, either aloud or silently. The teacher should then proceed to ask questions (not to make statements) about the explanation in order to see if the pupil has grasped the important ideas. The teacher should try *not* to answer a question the answer to which can be found in the book, but rather to ask the pupil to read aloud from the book the particular lines that will answer the question. Naturally the pupil would much rather listen to the teacher than to study the text, but the pupil can slowly be taught to see that the book does answer his questions. The pupil may be asked to close the book and tell what he has read. Or, a pupil may be asked to begin a problem, and then, when the pupil says he does not know what to do next, he may open the book and examine the explanation once more. The pupil who says he does not understand *any* of the explanation should be asked to read and state the meaning of one sentence at a time until he is able to state his specific difficulty. As a result of this method the pupil will feel more inclined to study the text further at others times, the pupil is trained to get information from a printed page, and we have the beginnings of self-reliant work. The pupil is learning something more than mere algebra.

We frequently hear the statements that some pupils learn more quickly by hearing than by reading—that some pupils learn through the eye and some through the ear. In the laboratory sciences, in particular, experiments have been made to determine which of the two methods is the better. Regardless of which is the better, it is undoubtedly true that when the pupil leaves school he must learn by studying a book. Hence we should teach him not merely algebra, but teach him how to get information from a book.

We also frequently hear the statement that the vocabulary of the subject is beyond the pupil's comprehension. This may be true of some of the older books in which the explanations were

written only for the teacher; but books have been improved and are now written for the pupil. When the explanations are read and studied in class, the pupil and the teacher have an opportunity to discuss the vocabulary and explain the new words and phrases. The so-called verbal problems, in particular, should not be assigned for homework without first being read in class. Frequently I ask for definitions of words, and I even go to the extreme of asking what is the subject and verb in a sentence, what noun is modified by this or that adjective or prepositional phrase. This is a simple way of curing a pupil's difficulty with the vocabulary.

We might say then as a fundamental theorem: All work dealing with the development of a subject should be done in class. This statement sounds so much like a stale platitude, so very obvious, that I need to explain what I mean by the development of the subject. I mean not merely the directions for working a new type of problem or performing a new operation, but also the work of showing how this new type of problem arises and how it differs from other problems. Under the old fashioned method the teacher would write two linear equations on the blackboard and then show how x and y are found, but there was little explanation of where the equations came from. The better way is to let the pupil see first a problem whose solution requires the study of two equations. After the equations have been developed from the problem, after the pupil has seen how this problem differs from earlier problems, then we may begin the work of solving the equations. What I have called the fundamental theorem may also be stated as: There never should be anything new in the homework. The place for the new work is in the classroom.

Perhaps one reason why the explanations in the book were not read by the pupil or were not studied in class was due to the fact that the explanations were too long. There is not time to read in class a thirty-minute explanation. But this trouble has also been eliminated. By breaking our work into smaller units we can have an explanation that can be studied in ten minutes or less. Take the matter of long division for example; the rules alone used to fill a page. Nobody ever read them. No one could ever learn long division by reading the rules in the

book. But the entire topic can be presented in half a page which the pupil can study in ten minutes.

When the explanation has been studied in class and the pupils have tried some problems, the teacher is ready to assign the homework for the next day.

When discussing the question of homework, the point is frequently made that homework is of little value because the pupils must invariably call upon other members of the family to help them do it. Also, mother is too busy with her clubs and bridge parties to do the homework, and father is too busy cleaning the car or fixing the radio. My contention is that if the pupil finds it necessary to call on others for help at home then the teacher has insufficiently prepared the pupil. The homework should consist *only* of such work as the pupil *can do* and knows *how to do*, but which he needs to do more accurately and speedily. Its object is to acquire skill in a *known* process—known and understood by the pupil. A pupil can not delegate this practise to someone else or ask someone else to help him with it.

The next day at the beginning of the period, the teacher begins the diagnostic work. As its name implies, the diagnosis consists in finding out how well the pupils have prepared the day's assignment, what their troubles and errors have been, and how ready the pupils are for the next step. We must admit that any work that the pupil has done at home may of course not be the pupil's work or not always a fair criterion of the pupil's ability. My own method consists in sending some pupils to the blackboard to work the problems that were assigned for homework. While they are at the board, I ask the other pupils to solve a new problem of the same kind. Then when walking from desk to desk I can see who is able to do the work, who has really done his homework, and what particular errors have given the most trouble. We thus have a kind of supervised study for about five minutes. If the preparation on the previous day has been careful, every pupil should be able to do most of the work. The only excuse for not having the work would be illness or a disinclination to do the work. The errors are due chiefly to carelessness, to speed, or lack of thoroughness in the work of some weeks past. Next, the class as a whole examines the work on the board. The pupils are told to compare their work with the work on the board with

a view to finding any errors. I aim to have the work at the board done by the brighter pupils so that it will be correct and so that I may give personal instruction to the weaker pupils at their seats. The chief reason for having this work on the board is to show the pupils some samples of good work, and to enable pupils to correct the errors on their homework papers.

Some teachers collect the homework at the beginning of the period. They justify this practise on the ground that pupils would otherwise surreptiously do in class the work that should have been done at home and that pupils even copy the work from the board. I can, however, easily notice as I walk around the room which pupils have not done the work, and the pupil can not copy any work from the board because I have given him some other work to do. But there is another reason why the homework should not be collected at the beginning of the period. If the pupil has made any mistake in his work, it is very important that the mistake be called to his attention, be corrected, and (most important) be corrected while the work is fresh. There are teachers, many teachers, who collect the homework at the beginning of the period, carefully mark and correct every error on the paper, grade the paper, record the grade, and return the paper to the pupil the next day. I shall not try to estimate for you how many hours it takes to examine 180 papers each day in this way, especially as it must all be done after the teacher has spent her full quota of assigned hours in school. But I believe that much of the value of this work is lost for two reasons: when the pupil the next day receives the corrected papers he has doubtless overcome the errors previously made; and second, in the intervening hours the pupil has lost interest in those particular problems. It is better to anticipate errors and prevent them than to correct them afterwards.[1]

Hence it is my practise to allow the pupils to retain their homework papers and to insist that they correct their errors while the subject is fresh. After they have corrected the work, the work is collected. If it is the first day's work on a new topic, I do not hesitate to throw the papers at once into the waste

[1] On the importance of anticipating errors rather than correcting them afterwards see "Repetition of Errors in Algebra," by Flora L. Scott, Mathematics Teacher, Feb. 1925, page 92.

basket. The first time I do this, there is much consternation among the pupils, and I tell them frankly that since they have done the work and corrected their errors, the homework has served its purpose and we are through with it. I surely do not wish to penalize the pupil for their errors; if the errors were numerous the pupils are not to blame but rather the teacher for not having properly prepared the pupils on the previous day. By that attitude I am trying to practise the same degree of honesty toward the pupil that we expect of him. Perhaps two or three times a week I check off in my class book the pupils who handed in the work, and the next day announce the names of the pupils who are failing to do the required work.

The next part of the day's work is devoted to drill. Some of the pupils are sent to the blackboard, and successive problems increase in difficulty. I prefer to say very little about the drill in order to leave with you the impression that under the present crowded conditions, the most important part of the teacher's work lies in the two previous steps; namely, preparing the pupil to do independent work without the teacher's help, anticipating his errors and correcting the errors before they can become habits. We know that drill consists chiefly in having the pupil work as many problems as possible, remembering that it is better to work a few problems correctly than a great many incorrectly. In part of this time I include the work of giving tests of various kinds, usually two each week of ten minutes each.

Although I have divided the period into three parts, the parts are not all of the same length. I suppose that at the end of the year, one half of the time has been devoted to correction and drill, and one half to the preparation and development necessary for the homework. There are doubtless many arguments that can be presented to show that this is not an ideal arrangement. My contention is merely that with large classes and as long as pupils can not be under the supervision of the teacher for a very large part of the day, it is the teacher's first duty to teach the pupil how to become independent of the teacher.

DISCUSSION

By P. STROUP
West High School
Cleveland, O.

In the Mathematics section of the Ohio State Educational Conference at Columbus, April 9, 1926, Miss Marie Gugle of that city read a paper on the above subject. A change suggested was that the extended teaching of intuitive geometry in the Junior High School be taken advantage of by spending less time on demonstrative plane geometry in the tenth year and thus make room for solid geometry as a part of that year's work. The high school course would then be completed with the customary algebra, some analytics and trigonometry.

The general movement to press the mathematics courses farther and farther down in the schools cannot go on without limit. Such a movement is likely to be of the oscillating kind and swing past the most desirable point. I am under the impression that it has already swung past that point. There is evidence of this in the large percent of failures in ninth grade algebra even with the drilled in method which is resorted to to alleviate the difficulty. There is evidence in the resort to arithmetic courses in the ninth grade to while away the time until more of the students quit or attain sufficient mental growth to handle the abstractions of algebra. One reason for the creation of the Junior High School was the fact that the students were merely marking time in the old eight grade. Yet Schorling finds that only 35.8% of them can write .4 as %. He says that in general they are passed on a mark of 70 yet he finds difficulty thinking of a set of questions on which they could really attain a mark of 70. The attempt to show algebra students that we use the same fundamental truths in dealing with algebraic expressions as with the Arabic system falls down because they do not appreciate the internal workings of that system. They have never had the cover off and watched the wheels go around.

The suggestion seems obvious that the field which is admittedly theirs should be more intensely cultivated before taking other than sight seeing tours in the adjacent territory. There is some justification for the intuitive geometry as a field for the practical application of arithmetic and the extention of useful vocabulary. But even in this the question arises whether it is not ill advised to convert geometry into a subject of the "poured in" type. The

most serious difficulty that we have with the ninth grade algebra students is to get them interested in the why. Would it help this attitude as much to be told the facts of geometry as to find out the why of the Arabic system including decimals? A most illuminating adventure is to attempt to keep simple accounts or some other exercise with the Roman system.

Now comes the proposition that these students because they have acquired some geometric vocabulary can therefore by elimination and compression reduce the time on plane geometry sufficiently to squeeze solid geometry into the same year. It sounds as reasonable as suggesting that because a class has learned the names of the apparatus in the gymnasium they could greatly reduce the time necessary to acquire a group of gymnastic accomplishments. It would help some but is it reasonable to expect that much of it. In the March, 1926 number of the Mathematics Teacher, page 154, Mr. Haerter concludes that since the instruction in intuitive geometry is found to be successful (by him) therefore we can only take advantage of the different preparation of the tenth grade students by extending the work to include solid geometry or trigonometry. There is a possibility that he overlooks. It is the doing of better work in the tenth grade. More orginals for the brighter students. When the upper third in the class does all the problems in the book, then it would begin to be time to take on a new block of work for the entire group. It may be that the people who can get solid geometry in the tenth year are the only ones who should go to college but the fact is that many others are going. Also there are many in our classes who are not going to college. Such an accelerated course can only have a limited usefullness and those who can follow it would be better prepared for college mathematics by becoming more sure of themselves in the usual requirements. The colleges are asking us to prepare the students for their work not to do it for them.

To return to the place where all speakers on education used to begin, the word educate means to lead out and not merely to fill. Mr. Bagley finds the basic cause of the crime wave in this country in the inefficient work of the elementary grades. He suggests less work better done. Would not the same suggestion be better for the high schools than the opposite tendency?

NEW NOTES

The report of the National Committee, entitled, "The Reorganization of Mathematics in Secondary Education," is no longer available either from the Dartmouth College or from the office of Professor J. W. Young, Chairman of the Committee.

The sixth annual meeting of the Inland Empire Council of teachers of Mathematics was held at the Lewis and Clark High School, Spokane, Washington, April 7th and 8th. Miss Edith Greenberg, President, called the meeting to order.

Miss Helen Dunn of Waterville gave a paper on "The Teaching of Mathematics in the Junior High Schools." Miss Dunn has had experience in this line and was able to tell us of the aims of the subject.

Mr. Harold Bliss explained some of his devices for the improving of the teaching of Algebra in the smaller high school. In his classes the students are classified according to their I. Q. and more work is required of the brighter student. Also the student is allowed to complete the course as soon as he can and he is allowed to take up higher work or to put his time on other school work.

Miss Mildred Hunt of Cleelum, Washington, told of some of the difficulties which arise in the Geometry class and her solution of these problems.

Miss Faith Fassett told of her experiences with the Standardized Algebra tests. She emphasized particularly the diagnostic value to the inexperienced teacher.

The Council has been doing a quite valuable piece of work under the direction of Prof. Walter C. Eells, of Whitman College, Walla Walla, Washington. The standardized Algebra tests were given in many of the High Schools of the state and the results sent in to Mr. Eells who then tabulated his results. This past year the same thing has been done in Geometry. These tests are not so easily given and as a result not so many schools sent in reports to Mr. Eells. Nevertheless he was able to work out a very interesting interpretation of the results.

The council has been greatly indebted to Professor Eells for the time he has spent on this work and they feel that this is the best piece of work that has been done for us for sometime.

The Joint Meeting of the Men's and Women's Mathematic Clubs of the City of Chicago was held on the evening of March 19th at the Great Northern Hotel. Mr. Jos. A. Nyberg, Chairman of the Program Committee, had arranged a unique display of "mathematical posters" around the walls of the banquet hall. Each poster hid within its artistry the meaning of a mathematical term. As the guests entered the hall they were given a score sheet and were informed that the one solving the most posters would be given as a prize one volume of Prof. David Eugene Smith's "History of Mathematics," the remaining volume to be given as second prize. At 9:45 P. M. a delicious banquet was served. President Marx Holt introduced as the speaker of the evening, Miss Genevieve Melody, Principal of the Calumet High School, Chicago, who spoke on the subject: "Impressions of a High School Principal." Following this address the winners of the poster contest were determined. Lake View High School of Chicago won first prize and Proviso Township High School of Maywood won second prize.

Mr. C. M. Austin, Oak Park, chairman of the Year Book Committee, made a short talk on the merits of the 1st year book and as a result sold a goodly number of copies of the National Council's new publication.—EDWIN W. SCHREIBER

NEW BOOKS

Standard Service Arithmetics—Book Two—By Knight, Studebaker and Ruch; Scott, Foresman and Co., 1926.

In Book Two of their Standard Service Arithmetics, Knight, Studebaker and Ruch have made an invaluable contribution to the teaching of arithmetic in grades five and six.

The material has been carefully graded so as to introduce only one new difficulty at a time and each new difficulty is prepared for by reading lessons, questions, and examples worked out as illustrations to insure understanding before actual drill is begun. The drill itself has been admirably arranged not only to provide for initial learning, but, by frequent and uniform distribution, to avoid the forgetting of previous instruction and provide for relearning.

Besides the matter of explanations and drills which go to make up the mechanical aspect of arithmetic the authors have made ample provision for the development of habits of self-criticism and self-reliance which make for the growth of power rather than mere skill. The Progress Chart, in the back of the book, which the child keeps himself provides him constantly with a record of his successes and failures and motivates extra drill on the weak spots. Such recurring ideas as "If *you* are not pleased with your work on the quick review ask your teacher for extra work" put the responsibility on the child. Self reliance is encouraged by the provision made for self-checking and immeasureably augmented by the student's Self-Help Index, an excellent device.

Besides all these features anyone of which would make the book a notable one the authors have been particularly happy in their choice of vocabulary. They have been most successful in their attempt to adapt material to the mind of the child in diction as well as organization.

Although the authors have failed to get away entirely from the hampering of tradition, especially in their treatment of fractions and decimals, on the whole the book is thoroughly in accord with modern educational theory. (JOSEPHINE HALEY.)

Zeitmessung und Sterndeutung in geschichtlicher Darstellung—By Professor Drecker; Berlin, Gebrüder Borntraeger, 1925, pp. 188.

This is Band 8 of the Sammlung Borntraeger, a series begun in 1923 and devoted to the field of the natural sciences. The purpose of each volume is to give to students in brief form a general survey of some special branch of science. In this particular case the branch is a divided one, the first part relating to chronology and the typical calendars of the world.—to the methods of measuring the time of the year; the second, to horology,—to the methods of measuring the time of the day; and the third to astrology,—to the method of measuring the influence of the stars upon human life.

Part I contains a survey of the history of the measures of the year, the month, and the week, with a discussion of various eras. The subject has been so often discussed in the more elaborate books upon chronology (in particular, F. K. Ginzel's *Handbuch der . . . Chronologie*, 3 vols., Leipzig, 1906-1914) that this survey, well-written though it is, can hardly be said to be more than an introduction to the subject.

Part II gives a very readable discussion of the leading types of sundials, waterclocks, and stellar clocks and is illustrated in a helpful way. While not pretending to do for the subject what the student will find done in the monumental work edited by Dr. Ernst von Bassermann-Jordan (*Die Geschichte der Zeitmessung und der Uhren*, Berlin and Leipzig, 1920-), it nevertheless will be found very helpful to those who wish a brief but scholarly introduction to the theory.

Part III gives a survey of the general nature of astrology with a considerable amount of interesting material relating to the rise and fall of faith in the theory. To the student of the history of mathematics it has the added interest that it gives the horoscope of Adriaen Metius, whose name is connected, though rather backhandedly, with the value of π. It seems that he was born at 6 P. M., on Dec. 9, 1571 (O.S.), in latitude 52° 50′, from which the horoscope is worked out to the astrologer's satisfaction.

As stated above, the book may be recommended as a scholarly introduction to the subjects discussed.

DAVID EUGENE SMITH

The Geometry of Rene Descartes—Translated from the French and Latin by David Eugene Smith and Marcia L. Latham with a Facsimile of the first edition, 1637, Chicago. The Open Court Publishing Company, 1925. Pp. XIII + 246. Price $4.00.

The preface to this work calls attention to the fact that "the epoch-making treatise of Descartes has never been printed in our language, or, if so, only in some obscure and long-since forgotten edition," and also claims that "certainly a work of this kind ought to be easily accessible to American and British students of the history of mathematics, and in a language with which they are entirely familiar."

The book is set up with the facsimile on one page and the translation in general facing it on the opposite page. This enables the reader to consult easily the original and to make those comparisons which add to the zest and satisfaction of reading a work of such intense interests to any student in the field of mathematics. Copious notes accompany the translation. These afford an elucidation of the text and add a fund of information bearing on it, besides furnishing further sources of material relating to the general subject. There is thus provided a history of the development of analytic geometry. The teacher of college algebra will find here the original of much that is presented in the section on theory of equations.

The Open Court Publishing Company has added to the long list of its notable publications in making possible such a work as this and the translators of it have put workers in the field of the history of science greatly in their debt.

Lao G. Simons
Hunter College

Modern Junior Mathematics—Book Two. By Marie Gugle. Gregg Publishing Company, 1926. Pp. 339.

The revision includes practice problems by chapters, shop and home economics problems, a series of tests organized by chapters and topics and a chapter on the relation of mathematics and art.

New Second Course in Algebra. By Herbert E. Hawkes, William A. Luby and Frank C. Touton. Ginn and Company, 1926. Pp. 333. Price $1.28.

This book is a revision of the authors' Second Course in Algebra and follows their new First Course in Algebra. It is brought out in a brief edition to provide for a half year's work beyond the work usually done in the first high school year, and in an enlarged edition to provide for those schools in which a full second year is given to algebra. The brief edition conforms to the recommendations of most of the examining bodies. It includes chapters on Determinants, Variation and Trigonometry.

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